past the leading edge does not separate can be approximately realized. To confirm the above, the distance from the trailing edge to the dipole $\bar{d}$ and to the critical point $\bar{s}$ in the limiting case versus α are shown in Fig. 5; both are related to the same quantities for the case of $\alpha = 90°$. Consequently, even for $\alpha \neq 90°$, the pattern of the discrete formation of the first (initial) vortex for the developing circulatory flow in an inviscid, incompressible fluid is retained.

However, the formation of the second vortex at the trailing edge of the plate cannot be described by employing only the dipole model. In order to obtain the correct pattern of the flow near the trailing edge after bifurcation occurs, it is necessary to realize that the amount of the fluid separated from the plate has already been acting on the flow as a vortex with circulation Γ_1. This strongly complicates the description of the flow. However, if one assumes that the second vortex will be formed only after the first one moves sufficiently far downstream, then the dipole model allows one to obtain an estimate of the circulation for the second vortex. For the plate with $\alpha = 90°$, $\Gamma_2 = 0.1631\Gamma_\infty$.

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DISPLACEMENT OF THE FREE SURFACE OF A FLUID DURING FLOW OVER A CYLINDER

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UDC 532.5

As is well known, the problem concerning potential flow over a cylinder by a fluid with a free surface may be reduced to integral equations [1], the exact solution of which is not known. In practice, much attention has been given to an approximation due to Lamb [1] in which the cylinder is replaced by a hydrodynamic dipole and the problem is then solved in a linear setting. Our purpose here is to provide experimental verification for Lamb's approximation and further perfection of his theory (taking into account the nonpotential nature of the flow over the cylinder).

An experiment was conducted in a Plexiglas trough (150 × 50 × 18 cm) in which a cylinder of radius R = 1 cm was moved at various speeds U (from 20 to 80 cm/sec) in a horizontal direction. As a rule, the full depth of the fluid was several times the depth h of immersion of the cylinder, so that the influence of the trough bottom was insignificant; Froude number $Fr = U^2/gh$ did not exceed 3. The free surface profile was studied photographically with the help of a scaling grid marked on a side wall of the tank; in addition, a conductivity data unit was used for sufficiently small displacements (of order less than 1 mm).

The form of the surface above the cylinder depends essentially on Fr; its form for Fr = 2 is shown in Fig. 1. When Fr > 1 a bulge forms above the cylinder, its maximum corresponding to the center coordinate of the cylinder; when Fr is decreased (Fr < 1.2-1.3) the bulge diminishes and is displaced forward. For small Fr (Fr = 0.3-0.5) bulges above the body are generally not observed; the water level goes down smoothly, going into a depression behind the cylinder. In the region behind the cylinder one observes a rapidly diminishing (of at most 1-2 periods) surface wave propagating with velocity U.

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Fig. 1

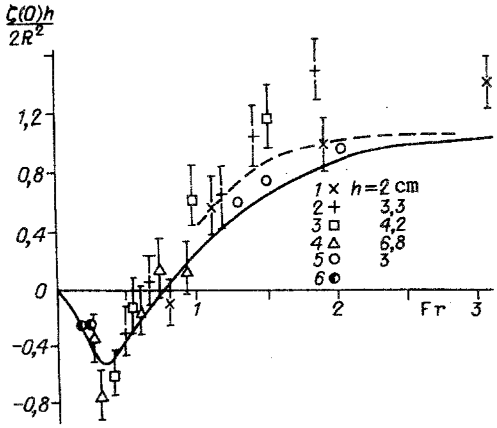

Fig. 2

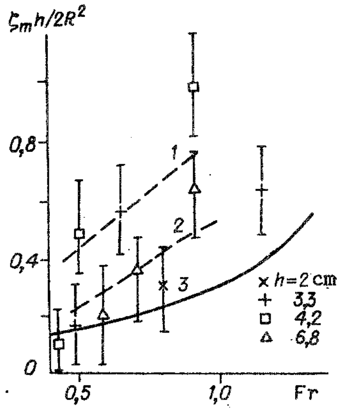

Fig. 3

We present first the known facts concerning generation of a wave by a moving source in the Lamb approximation. Displacement of the fluid suface in a linear formulation is determined from the expression

$$\frac{\zeta h}{2R^2} = (1+\tau^2\mathrm{Fr}^2)^{-1} - \int\limits_0^\infty \frac{\sin t - \mathrm{Fr}\, t\cos t}{t^2\mathrm{Fr}^2+1} e^{-\tau \mathrm{Fr}\, t}\, dt + \frac{\zeta_\sim h}{2R^2}, \tag{1}$$

where $\tau = gx/U^2$ is a dimensionless coordinate corresponding to the direction of the cylinder; $\zeta_\sim$ is the wave part of the surface displacement: $\frac{\zeta_\sim h}{2R^2} = \frac{2\pi}{\mathrm{Fr}} e^{-1/\mathrm{Fr}} \sin\tau$. The quasistatic part, consisting of the first two terms in Eq. (1), is easily found in the two limiting cases of small and large Froude numbers; in addition, the wave part tends towards zero:

$$\frac{\zeta h}{2R^2} = \begin{cases} \mathrm{Fr}\,\dfrac{\mathrm{Fr}^2\tau^2-1}{(\mathrm{Fr}^2\tau^2+1)^2} & \text{for} \quad \mathrm{Fr} \ll 1, \\ \dfrac{1}{1+\mathrm{Fr}^2\tau^2} & \text{for} \quad \mathrm{Fr} \gg 1. \end{cases}$$

For intermediate values of Fr the quasistatic part ζ is handled numerically. When Fr is of order 1 the wave part is substantial; its amplitude a can be several times larger than the quasistatic part.

The picture of the surface profile becomes even more complicated when the nonpotential flow is taken into account. A turbulent wake forms behind the moving body and plays the role of an additional source of mass (a monopole) [2]. As can be readily verified, the monopole yields an additional displacement of the surface:

$$\frac{\zeta_m h}{2R^2} = \begin{cases} \dfrac{c_x h}{2\pi R}\mathrm{Fr}\,\dfrac{\mathrm{Fr}\,\tau}{1+\mathrm{Fr}^2\tau^2} & \text{for} \quad \mathrm{Fr} \ll 1, \\ \dfrac{c_x h}{2\pi R}\,\mathrm{arc\,tg}\,\mathrm{Fr}\,\tau & \text{for} \quad \mathrm{Fr} \gg 1. \end{cases}$$

Here c_x is a resistance coefficient; for Reynolds numbers Re ≈ 10^4, corresponding to the conditions of the experiment, $c_x \approx 1$. In addition, viscosity must lead to a damping of the waves, whereby a determining role, as we shall show, is played by the turbulent viscosity associated with the presence of a turbulent wake behind the cylinder.

We proceed now to a description of the results of our measurements. Figure 2 exhibits the theoretical dependence of the displacement of the water level at the point $\zeta(0)h/2R^2$, corresponding to the cylinder center, on Fr. Plotted here are the results of measurements of the displacements obtained from photographs (points 1-4) and from the data unit (points 5). The systematic amount by which the measured values $\zeta(0)$ exceed the theoretical values when Fr > 1 should be noted, an amount on the average of 50%. However, in the region where Fr < 1 we observe a satisfactory agreement of measured and theoretical results. Calculation of the nonlinear terms in the boundary conditions on the free surface, presented in [3] for small Fr values (points 6 for Fr = 0.16 and 0.22), leads to a raising of the level of the surface directly above the cylinder. When Fr > 1, local variation of Fr for flow over a cylinder [4] can have a noticeable influence on perturbations in a neighboring zone (by 20%

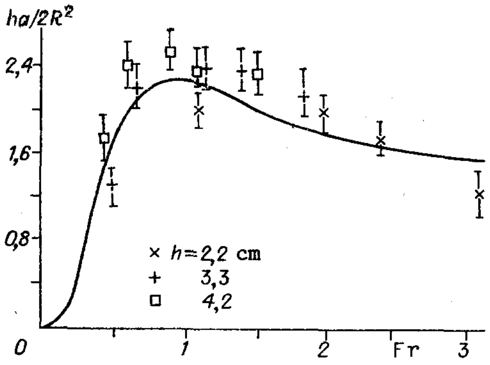

Fig. 4

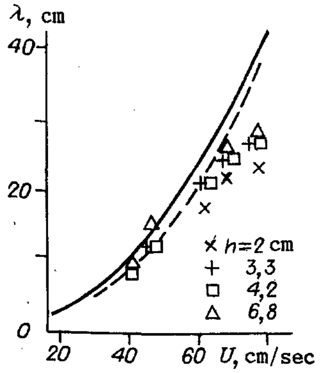

Fig. 5

and 50% for h = 6.8 and 2 cm). Calculation of this variation leads to a displacement of the theoretical curve to the left (the dashed curve for h = 3.3 cm) and to a better agreement of theoretical and experimental data.

Figure 3 gives the results of measurements ζ_m of the bulge, for 0.5 < Fr < 1.2, displaced somewhat ahead of the cylinder center. However, here the theoretical curve 3 ($\zeta_m h/2R^2$) was constructed taking into account only the dipole source. As can be seen, divergence of the measured and theoretical values is fairly substantial (twice on the average). Taking the monopole term into account decreases this divergence somewhat (curves 1 and 2 for h = 6.8 and 2 cm).

As for the rolling wake behind the body, we note here the good correspondence of the depression behind the body with the theoretically calculated amplitude for all values of Fr > 0.5 (Fig. 4). Values of the characteristic wave length lie, for small velocities, on the curve $\lambda = 2\pi U^2/g$ (Fig. 5); however, with an increase in U (U > 40 to 50 cm/sec) one observes a tendency for λ to decrease in comparison with theoretical values. Taking into account the nonlinear correction [5] $\lambda_1 = \lambda(1 - 4\pi^2a^2/\lambda^2)$ for $2\pi a/\lambda \ll 1$ (the dashed curve for h = 4.2 cm) leads to an improvement in the correspondence of theoretical and experimental values. The wave amplitude, according to theory, is constant in the two-dimensional case; in the experiment, however, one observes a wave that dies out rapidly (in 1 to 2 oscillations). This strong damping of the wave in the experiment is due, in our opinion, to damping by turbulence in the wake behind the cylinder. Actually, estimating the turbulent viscosity $\nu \sim ul$ ($u \sim \sqrt{F_x/\rho x}$ is the mean velocity of the turbulent motion in the wave; $l \sim \sqrt{xF_x/\rho U^2}$ is the halfwidth of the wake; $F_x = \rho U^2 c_x R$ is the resistance force, plotted per unit cylinder length in [2]), we obtain $\nu \sim F_x/\rho U \sim \text{const}$.

Defining the damping length L as the distance over which the wave amplitude undergoes an e-fold decay, we have $L \sim g^3/(2\omega^5 c_x UR) = \lambda^2/(8\pi^2 c_x R)$. When the velocity varies from 40 to 80 cm/sec, we find that the ratio L/λ varies from 0.1 to 0.5, i.e., the quenching action of turbulence in the wake behind the cylinder is very large.

Thus, for not very large Froude numbers, the motion of the cylinder is accompanied by a displacement of the surface close to the cylinder. A theoretical description of this process may be obtained by introducing potential and force sources. Taking viscosity into account is of special importance in analyzing the waves studied. Similar results were obtained in [6] in a study of internal waves, which, in contrast to our case, are weakly absorbed by the turbulent wake.

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